Renormalization of the Minimal Supersymmetric Standard Model

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The renormalization of the Minimal Supersymmetric Standard Model (MSSM) is presented. We describe symmetry identities that constitute a framework in which the MSSM is completely characterized and renormalizability can be proven. Furthermore, we discuss applications of this framework for the determination of symmetry-restoring counterterms, the gauge dependence of $\tan\beta$ and the derivation of non-renormalization theorems.

In this talk the renormalization of the Minimal Supersymmetric Standard Model (MSSM) is presented [1]. A framework is set up where all counterterms are uniquely determined. This comprises a set of symmetry identities providing a complete characterization of the MSSM and a set of on-shell renormalization conditions that forbid on-shell mixing between different physical fields. In this framework it has been shown that the MSSM is multiplicatively renormalizable, infrared finite, and that all on-shell conditions can be satisfied simultaneously.

This study is motivated by the fact that no satisfactory regularization for supersymmetric gauge theories is known. In particular, dimensional regularization breaks supersymmetry; hence, supersymmetry-restoring counterterms have to be calculated and added. On the other hand, dimensional reduction is mathematically inconsistent, and therefore its area of validity is unclear. In practice, renormalization of the MSSM means first to check whether the chosen regularization preserves the symmetries and to add — if necessary — symmetry-restoring counterterms, and second to add the usual symmetric counterterms (corresponding to field and parameter renormalization) in order to cancel divergences and satisfy renormalization conditions.

This raises the deeper question of how to formulate the symmetries of the MSSM at all on the quantum level and on the level of Green functions. For the Standard Model [2] and general supersymmetric Yang–Mills theories [3,4] the answer is known: the Slavnov–Taylor and Ward identities provide a complete characterization of the symmetries. Similar identities should also be formulated in the MSSM.

The outline of the talk is as follows. First the symmetry identities of the MSSM are presented. Then we draw important conclusions of these identities, in particular on the proof of renormalizability and the practical determination of symmetry-restoring counterterms. Finally, two applications of the symmetry identities are discussed, concerning the gauge dependence of the parameter $\tan\beta$ and a new approach to the non-renormalization theorems in supersymmetric gauge theories.

The basic symmetries of the (electroweak part of the) MSSM are spontaneously broken SU(2)×U(1) gauge invariance and softly broken supersymmetry. Clearly, the basic structure of the symmetry identities in the MSSM can be obtained by combining the results for the Standard Model [2] and for general supersymmetric Yang–Mills theories [3,4]. The main symmetry content is described by a Slavnov–Taylor identity

$$S(\Gamma) = 0, \tag{1}$$

where Γ denotes the generating functional of the one-particle irreducible Green functions. It combines all information on gauge invariance and supersymmetry including the quantum corrections to the transformations and the commutation relations of the generators. However, the Slavnov–

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Taylor identity does not fix the values of the hypercharges Y_i, which however are crucial in order to fix the electric charges $Q_i = T_i^3 + Y_i/2$ correctly. The Y_i are fixed by a local Ward identity for the U(1)-symmetry:

$$\partial^\mu \frac{\delta \Gamma}{\delta V'^\mu} = -ig' \sum_{\text{Fields } \varphi_i} \frac{Y_i}{2} \, \varphi_i \frac{\delta \Gamma}{\varphi_i} + \begin{array}{c}\text{gauge-fixing}\\ \text{terms}\end{array}$$

Along with this local Ward identity, global Ward identities describing global SU(2)×U(1) and R invariance are formulated. The symmetry breakings are introduced by using external fields with a constant shift. For soft supersymmetry breaking a chiral supermultiplet $(A, a_\alpha, F_A + v_A)$ with a shift in its highest component is used, and for spontaneous breaking of gauge invariance an SU(2)×U(1)-multiplet $(\hat{\Phi} + \hat{\mathrm{v}})$ is used. Using these fields, the symmetry identities take the same form as in the cases with unbroken symmetries, but when the external fields are set to their constant values, symmetry breaking is described.

While the basic structure of the symmetry identities seems obvious, the difficulty lies in the detailed implementation. In fact, it turns out that the R_ξ-gauge requires that in the MSSM the detailed structure of the external fields $(\hat{\Phi} + \hat{\mathrm{v}})$ appearing in the symmetry identities must differ from the one in the literature.

The problem is that the MSSM contains an extended Higgs sector, and even if CP-conservation is assumed the physical CP-odd Higgs boson A^0 and the charged Higgs bosons $H^\pm$ can mix with unphysical degrees of freedom:

$$\begin{array}{rcl} A^0 & \leftrightarrow & (G^0, A^\mu_{\text{Long}}, Z^\mu_{\text{Long}}), \\ H^\pm & \leftrightarrow & (G^\pm, W^{\pm\mu}_{\text{Long}}). \end{array} \tag{2}$$

However, in the R_ξ-gauge fixing terms only the unphysical fields appear in the gauge-fixing functions (we restrict ourselves to the case of the neutral fields for simplicity):

$$F^A = \partial^\mu A_\mu, \quad F^Z = \partial^\mu Z_\mu + \xi M_Z G^0. \tag{3}$$

In this form, the gauge fixing would even break global SU(2)×U(1)-invariance and the U(1)-Ward identity. As proposed in [5,2], the U(1)-Ward identity can be restored by using the external field $(\hat{\Phi} + \hat{\mathrm{v}})$ for writing the gauge-fixing functions. However, if $(\hat{\Phi} + \hat{\mathrm{v}})$ is chosen as an SU(2)-doublet like in the Standard Model-case, it turns out that necessarily A^0 and/or $H^\pm$ appear in the gauge fixing. Thus we are lead to the question which multiplet assignment to choose for $(\hat{\Phi} + \hat{\mathrm{v}})$. The answer has been found in [1]. The multiplet structure of $(\hat{\Phi} + \hat{\mathrm{v}})$ is chosen as the product of the adjoint and doublet representation of SU(2)×U(1), and there are two of these 8-component multiplets, one for each Higgs doublet $H_{1,2}$. Then the gauge-fixing functions can be written as

$$F^a = \partial^\mu V^a_\mu - 2\mathrm{Im}((\hat{\Phi} + \hat{\mathrm{v}})_i^{a\dagger} H_i), \tag{4}$$

so that F^a transforms in the adjoint representation and is compatible with the U(1)-Ward identity. At the same time, $\hat{\mathrm{v}}$ has enough components that can be adjusted such that the R_ξ-gauge conditions are reproduced for $\hat{\Phi} = 0$, i.e. $F^a|_{\hat{\Phi}\to 0}$ does not contain the fields A^0, $H^\pm$, and the $F^{A,Z}$-components coincide with (3).

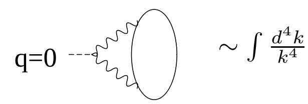

Figure 1. Infrared divergences originating from Ward identities $W\Gamma = \ldots + \int d^4x \hat{\mathrm{v}}_i \frac{\delta\Gamma}{\delta\hat{\Phi}_j}$ if counterterms like $\mathcal{L}_{\text{ct}} = \hat{\Phi}_j A^\mu A_\mu$ are present.

A more complicated multiplet structure of this external field, however, also complicates the proof of infrared finiteness of the MSSM. Consider the counterterm $\mathcal{L}_{\text{ct}} = \hat{\Phi}_j A^\mu A_\mu$ that might be necessary to restore symmetries as an example. In the calculation of global Ward identities there appear terms like $W\Gamma = \ldots + \int d^4x \; \hat{\mathrm{v}}_i \frac{\delta\Gamma}{\delta\hat{\Phi}_j}$, leading to diagrams of the type shown in Fig. 1. Since the $\hat{\Phi}_j$-field carries no momentum, all diagrams of this type contain the infrared divergent integral $\int \frac{d^4k}{k^4}$, no matter what other external lines and momenta are present. It has to be shown that such situations cannot arise and that Γ itself as well as all Ward and Slavnov–Taylor identities are infrared finite. The proof of infrared finiteness can be carried out by identifying the dangerous components of $\hat{\Phi}$ and also of $(A, a_\alpha, F_A + v_A)$ using infrared powercounting, finding an optimal assignment of the infrared quantum numbers and checking that every dangerous term is in fact excluded for a symmetry reason.

Combining the mentioned elements yields the detailed structure of the symmetry identities in the MSSM. The main identities are the Slavnov–Taylor and Ward identities in presence of the external $(\hat\Phi + \hat{\mathrm{v}})$ and $(A, a_\alpha, F_A + v_A)$ fields, whose multiplet structure and infrared powercounting has been determined. There are further identities corresponding to gauge-fixing conditions, but their discussion is beyond the scope of the present talk (see sec. 3.2 of [1]). The set of symmetry identities constitutes a full definition of the MSSM at the quantum level.

Using the symmetry identities we can draw the following conclusions of basic importance: the symmetry-restoring counterterms are uniquely determined, the remaining freedom consists of adding symmetric counterterms — which can be shown to correspond to multiplicative renormalization, and infrared finiteness can be completely proven.

Furthermore, the Slavnov–Taylor identity can be used to show that all on-shell renormalization conditions can be satisfied simultaneously [1]. Since the MSSM is multiplicatively renormalizable, it is quite obvious that most on-shell conditions can be satisfied be choosing the field renormalization constants appropriately. An exception arises from the mixings (2). The question is whether it is possible to establish on-shell renormalization conditions that characterize the fields A^0, $H^\pm$ as mass eigenstates. Using field renormalization it is always possible to satisfy the on-shell conditions (neglecting the finite widths of the particles)

$$\begin{aligned} &\Gamma_{A^0A^0}(M_A^2) = \Gamma_{A^0G^0}(M_A^2) = 0, \\ &\Gamma_{H^+H^-}(M_{H^\pm}^2) = \Gamma_{H^+G^-}(M_{H^\pm}^2) = 0. \end{aligned} \tag{5}$$

The Slavnov–Taylor identity yields

$$\begin{aligned} 0 &= \sum_{\varphi=A^0,G^0} \Gamma_{c^a Y_\varphi}\Gamma_{A^0\varphi} - \Gamma_{c^a\bar{c}^b}\frac{1}{\xi}\frac{\delta F^b}{\delta A^0} \\ &\quad + \sum_{V=A,Z} \Gamma_{c^a Y_{V_\mu}}\Gamma_{A^0 V_\mu}, \end{aligned} \tag{6}$$

and a similar identity for $A^0 \to H^\pm$, where $c^a, \bar{c}^a$ are the Faddeev–Popov (anti-)ghosts and Y_φ denotes the source of the BRS transformation of φ. Because of (5) and because the R_ξ-gauge can be realized, the first two terms of (6) vanish for on-shell momenta, and we obtain

$$\begin{aligned} &\Gamma_{A^0A^\mu}(M_A^2) = \Gamma_{A^0Z^\mu}(M_A^2) = 0, \\ &\Gamma_{H^+W^{-\mu}}(M_{H^\pm}^2) = 0. \end{aligned} \tag{7}$$

Hence, indeed no on-shell mixing between the physical and unphysical degrees of freedom occurs and all on-shell conditions can be satisfied.

This completes the general discussion of the renormalization and provides the basis for practical applications.

As the first application we will briefly discuss how supersymmetry-restoring counterterms can be determined in practice. Generally, the counterterms have to be chosen such that the symmetry identities hold after renormalization. For supersymmetry, to kinds of identities are important. Using[2]

$$0 = \frac{\delta S(\Gamma)}{\delta\epsilon} = \sum_{\text{Fields } \varphi_i} \frac{\delta\Gamma}{\delta\epsilon\delta Y_{\varphi_i}}\frac{\delta\Gamma}{\delta\varphi_i}, \tag{8}$$

where ϵ is the ghost corresponding to supersymmetry, we obtain identities corresponding to supersymmetry relations, where the prefactors are quantum corrected and renormalized supersymmetry transformations. Using

$$\begin{aligned} 0 &= \frac{\delta S(\Gamma)}{\delta\epsilon\delta\bar\epsilon\delta Y_{\varphi_j}} \\ &= \sum_{\text{Fields } \varphi_i}\left(\frac{\delta\Gamma}{\delta\bar\epsilon\delta Y_{\varphi_i}}\frac{\delta\Gamma}{\delta\epsilon\delta Y_{\varphi_j}\varphi_i} + (\epsilon\leftrightarrow\bar\epsilon)\right) \\ &\quad \pm 2i\sigma^\mu\partial_\mu\varphi_j + \dots\,, \end{aligned} \tag{9}$$

identities corresponding to the supersymmetry algebra are obtained. Here $\pm$ holds for bosonic/fermionic φ_j, respectively, and the dots denote calculable terms corresponding to gauge transformations and equations of motion in the supersymmetry algebra. By the requirement that

[2] There are fields φ with no corresponding source Y_φ. For them, $\frac{\delta\Gamma}{\delta\epsilon\delta Y_{\varphi_i}}$ has to be replaced by $\partial_\epsilon s\varphi_i$, where $s\varphi_i$ is the BRS variation of φ_i.

these identities should be satisfied after renormalization, it is possible to determine first the counterterms for the renormalized supersymmetry transformations, and then the counterterms to the vertex functions without BRS insertions. The results are unique up to symmetric counterterms, which correspond to multiplicative renormalization. Identities of the first kind have already been considered in [6], but they alone do not lead to unique results for symmetry-restoring counterterms and cannot serve as tests of the symmetry of a given regularization scheme. Both kinds of identities have been considered in [7,8] for supersymmetric QED and QCD at the one-loop level. In all cases it has been found that dimensional reduction preserves the identities at the regularized level. Thus this scheme is indeed supersymmetric in the cases considered up to now.

In the remainder of this talk we want to discuss two applications of the Slavnov–Taylor identities of the MSSM or other supersymmetric models: calculating the gauge dependence of the parameter $\tan\beta$ [9] and deriving non-renormalization theorems [10,11]. Both use certain extensions of the Slavnov–Taylor identities as tools. The gauge dependence is calculated using a Slavnov–Taylor identity containing an additional BRS transformation of the gauge parameter, and the non-renormalization theorems are derived by introducing BRS transformations of the coupling constants.

The quantity $\tan\beta$ is one of the main input parameters of the MSSM. At tree level,

$$\tan\beta = \frac{v_2}{v_1}. \tag{10}$$

In [9], different renormalization schemes for $\tan\beta$ were analyzed with the aim to find a scheme that defines $\tan\beta$ at the same time in a gauge-independent and process-independent way. Using the extended Slavnov–Taylor identity, the gauge dependence of $\tan\beta$ can be explicitly calculated for any given scheme. It was found that in several well-known schemes, the $\overline{DR}$-scheme and the schemes introduced in [12], $\tan\beta$ is defined in a gauge-dependent way, i.e. the relation between $\tan\beta$ and observable quantities that can be used to extract $\tan\beta$ from experiment is gauge dependent. The only exception is the $\overline{DR}$-scheme, which is gauge independent if its application is restricted to R_ξ-gauges and to the one-loop level.

Hence, a large class of new schemes was considered where $\tan\beta$ is defined in the Higgs sector, as at the tree level. All gauge-independent schemes in this class were identified. Unfortunately it turned out that each of these schemes leads to severe numerical instabilities in the perturbative expansion, so these schemes are not useful in practice. Given these results, the $\overline{DR}$-scheme seems to be the best choice of all process-independent definitions for $\tan\beta$.

Non-renormalization theorems are among the deepest and most exciting properties of supersymmetric models. They state the absence of certain divergences, e.g. of quadratic divergences, and thus provide in particular a solution to the naturalness problem. In [13], a new approach towards these theorems has been developed, which has been applied to supersymmetric QED and QCD in [10,11]. The origin of the non-renormalization theorems is identified as follows. Every term in a supersymmetric Lagrangian is the highest component of a supermultiplet and thus related to lower-dimensional field polynomials. Similarly, supersymmetry relates diagrams to diagrams with a lower degree of divergence.

This fact can be implemented into an extended Slavnov–Taylor identity by replacing the coupling constants by full supermultiplets. In this way, every supersymmetric term in the Lagrangian is replaced by a sum of the form (considering the case of chiral multiplets for simplicity)

$$g\mathcal{L}_{\rm susy} \to g\mathcal{L}_{\rm susy} - \chi\Xi + fA, \tag{11}$$

where (g,χ,f) is the multiplet of the coupling and $(A,\Xi,\mathcal{L}_{\rm susy})$ the multiplet of the respective term in the Lagrangian. The higher components of the coupling thus couple to the lower components of the Lagrangian. The supermultiplet structure of the couplings implies BRS transformations of the couplings. If these are included into the Slavnov–Taylor identity, extended identity is obtained that can be used to derive non-renormalization theorems.

The advantages of this approach are that the non-renormalization theorems can be derived without assuming a supersymmetric regularization, in the context of the Wess–Zumino gauge, and that it makes the underlying algebraic origin apparent. As an illustration, we list the results for supersymmetric QED. In this model, there are two independent divergences, conveniently expressed in terms of divergent renormalization constants for the charge and electron mass:

$$\delta Z_e^{(1)} : \text{ only one-loop}, \quad \delta Z_m^{(l)}. \tag{12}$$

The renormalization constants for all soft-breaking parameters can be expressed in terms of δZ_e and δZ_m:

$$\delta Z_{M_\lambda}^{(1)} = 2\delta Z_e^{(1)} : \text{ only one-loop}, \tag{13}$$

$$\delta Z_b^{(l)} = (2l M_\lambda m/b + 1)\delta Z_m^{(l)}, \tag{14}$$

$$\delta Z_M^{(l)} = \frac{1}{2} l(l+1)(M_\lambda^2/b)\delta Z_m^{(l)}. \tag{15}$$

In addition to relating all renormalization constants, these equations imply that the charge and the photino mass counterterms are finite from the two-loop level on and that the two scalar mass counterterms are only logarithmically divergent.

Similar results can also be derived for non-abelian supersymmetric gauge theories. In this case a deep connection between the form of the non-renormalization theorems and two anomalies — the Adler–Bardeen anomaly and a supersymmetry anomaly in presence of the supercoupling [11] — is exhibited. As a byproduct, also the non-renormalization of the Adler–Bardeen anomaly coefficient can be proven in a simple way.

To summarize, we started with a discussion of the symmetry identities of the MSSM. Particular attention was payed to the mixing of physical and unphysical fields and the implications on the external fields $(\hat{\Phi} + \hat{\text{v}})$, $(A, a_\alpha, F_A + v_A)$ and their structure. Once the detailed form of the fields and the symmetry identities was established, it was possible to prove the renormalizability of the MSSM. On the practical side, the symmetry identities constitute an important tool. We have shown that they can be used for the unambiguous determination of possible symmetry-restoring counterterms, for calculating the gauge dependence of $\tan\beta$, and in a new approach to the non-renormalization theorems.

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